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REFLECTIONLESS ABSORPTION OF ELECTROMAGNETIC RADIATION INCIDENT AT AN ANGLE UPON A CLARIFIED ABSORBING SUBSTRATE

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The conditions for the occurrence of complete reflectionless absorption of an electromagnetic wave when it is incident at an angle upon an absorbing substrate of infinite thickness with a layer of nonabsorbing dielectric applied to it have been found. The dependence of these conditions on the thickness of the layer of clarifying coating, angle of wave incidence, and the dielectric properties of the substrate and coating materials is investigated.

In [1] the possibility of complete or reflectionless absorption of plane-parallel electromagnetic radiation when it is incident normally on an absorbing dielectric substrate with an applied plane layer of coating from a nonabsorbing material was established theoretically. At a given radiation frequency complete absorption occurs at strictly defined values of the coating layer thickness and dielectric coefficients of the coating and substrate materials and by its results is analogous to the clarifying action of quarter-wave coatings for nonabsorbing substrates [2]. It is assumed that the indicated phenomenon may take place also in those cases where electromagnetic radiation is incident on the considered layered system at an angle to its surface. This assumption is supported by the theoretical investigations carried out in [3] that confirmed the possibility of complete absorption of electromagnetic radiation incident at an angle on a layer of an absorbing dielectric applied to a metallic substrate.

In order to solve the problem set, we consider incidence of a plane wave at an angle α_0 to the surface of a plane layer of nonabsorbing dielectric with permittivity ε_1 applied to an absorbing substrate of infinite thickness and having a complex value of permittivity $\hat{\varepsilon} = \varepsilon' - i\varepsilon''$. Here, with account for the position of the electric polarization vector **E** of the wave relative to the plane of its incidence, we will distinguish the cases of reflection of a parallel-polarized (PP) and transversely polarized (TP) waves, when the vector **E** is parallel to the wave incidence plane or perpendicular to it. Depending on the type of polarization of an incident wave, a complex expression for the coefficient of wave reflection $\hat{\rho}$ from the considered plane lamellar system has the form

$$\hat{\rho} = \frac{Z_{\text{in}} \cos \alpha_0 - Z_0 \cos \alpha_1}{Z_{\text{in}} \cos \alpha_0 + Z_0 \cos \alpha_1} \ \ (\text{TP}) , \quad \hat{\rho} = \frac{Z_0 \cos \alpha_0 - Z_{\text{in}} \cos \alpha_1}{Z_0 \cos \alpha_0 + Z_{\text{in}} \cos \alpha_1} \ \ (\text{PP}) , \tag{1}$$

where $\cos \alpha_0 = \sqrt{1 - p}$; $\cos \alpha_1 = \sqrt{1 - p/\varepsilon_1}$; $p = \sin^2 \alpha_0$ [4], whereas the input resistances Z_{in} of the coating–substrate system are defined as

$$Z_{\text{in}} = Z_1 \frac{Z \cos \alpha_1 + Z_1 \cos \alpha_2 \tanh (\gamma l \cos \alpha_1)}{Z_1 \cos \alpha_2 + Z \cos \alpha_1 \tanh (\gamma l \cos \alpha_1)} \ \ (\text{TP}) ,$$

$$Z_{\text{in}} = Z_1 \frac{Z \cos \alpha_2 + Z_1 \cos \alpha_1 \tanh (\gamma l \cos \alpha_1)}{Z_1 \cos \alpha_1 + Z \cos \alpha_2 \tanh (\gamma l \cos \alpha_1)} \ \ (\text{PP}) , \tag{2}$$

where $\cos \alpha_2 = \sqrt{1 - p/\hat{\varepsilon}}$; $\gamma = i2\pi\sqrt{\varepsilon_1}/\lambda$.

For the reflectionless absorption of the wave in the lamellar system considered there correspond the conditions

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$$Z_{in} = Z_0 \frac{\cos\alpha_1}{\cos\alpha_0} \quad (\text{TP}), \quad Z_{in} = Z_0 \frac{\cos\alpha_0}{\cos\alpha_1} \quad (\text{PP}). \tag{3}$$

Since $Z_1 = Z_0/\sqrt{\varepsilon_1}$, $Z = Z_0/\sqrt{\hat{\varepsilon}}$, then, subject to (2), we have

$$\begin{gathered} \frac{\sqrt{\varepsilon_1 - p} + i\sqrt{\hat{\varepsilon} - p}\,\tan 2\pi x}{\sqrt{\hat{\varepsilon} - p} + i\sqrt{\varepsilon_1 - p}\,\tan 2\pi x} = \sqrt{\frac{\varepsilon_1 - p}{1 - p}} \quad (\text{TP}), \\ \frac{\varepsilon_1\sqrt{\hat{\varepsilon} - p} + i\hat{\varepsilon}\sqrt{\varepsilon_1 - p}\,\tan 2\pi x}{\hat{\varepsilon}\sqrt{\varepsilon_1 - p} + i\varepsilon_1\sqrt{\hat{\varepsilon} - p}\,\tan 2\pi x} = \varepsilon_1\sqrt{\frac{1 - p}{\varepsilon_1 - p}} \quad (\text{PP}), \end{gathered} \tag{4}$$

where $x = \frac{l}{\lambda}\sqrt{\varepsilon_1 - p}$. The quantities ε', ε'', and ε_1 entering into Eqs. (4) are connected with the refraction index n and the factor of dielectric losses y of the substrate and with the refraction index of the wave n_1 of the coating by the well-known relation

$$\varepsilon' = n^2(1 - y^2), \quad \varepsilon'' = 2n^2 y, \quad \varepsilon_1 = n_1^2, \tag{5}$$

where $n = \lambda/\lambda_d$; $n_1 = \lambda/\lambda_{1d}$; $y = \tan\delta/2$; $\delta = \arctan \varepsilon''/\varepsilon'$. For convenience in the further discussion we introduce the notation

$$\bar{\varepsilon}' = \frac{\varepsilon' - p}{1 - p}, \quad \bar{\varepsilon}'' = \frac{\varepsilon''}{1 - p}, \quad \bar{\varepsilon}_1 = \frac{\varepsilon_1 - p}{1 - p}, \tag{6}$$

where by analogy with (5) $\bar{\varepsilon}'$, $\bar{\varepsilon}''$, and $\bar{\varepsilon}_1$ can be presented in the form

$$\bar{\varepsilon}' = \bar{n}^2(1 - \bar{y}^2), \quad \bar{\varepsilon}'' = 2\bar{n}^2\bar{y}, \quad \bar{\varepsilon}_1 = \bar{n}_1^2, \tag{7}$$

where $\bar{n} = \bar{\lambda}/\bar{\lambda}_d$; $\bar{n}_1 = \bar{\lambda}/\bar{\lambda}_{1d}$; $\bar{y} = \tan\bar{\delta}/2$; $\bar{\delta} = \arctan \bar{\varepsilon}''/\bar{\varepsilon}'$; $\bar{\lambda} = \lambda/\sqrt{1 - p}$; $\bar{\lambda}_d$, $\lambda_{1d} = \sqrt{\varepsilon_1 - p}$. Using these designations in Eqs. (4) and making transformations, we obtain

$$\tanh(\alpha + i\beta + i2\pi x) = A, \tag{8}$$

where the parameters α and β can be found from the equation

$$\tanh(\alpha + i\beta) = N(1 - iY). \tag{9}$$

Depending on the type of wave polarization, the parameters N, Y, and A entering into Eqs. (8) and (9) are defined by the following relations:

$$N = \frac{\bar{n}}{\bar{n}_1}, \; Y = \bar{y}, \; A = \frac{1}{\bar{n}_1} \; (\text{TP}), \; N = \frac{n^2\bar{n}_1(1 - y^2 + 2y\bar{y})}{n_1^2\bar{n}(1 + \bar{y}^2)}, \; Y = \frac{2y - \bar{y}(1 - y^2)}{(1 - y^2) + 2y\bar{y}}, \; A = \frac{\bar{n}_1}{n_1^2} \; (\text{PP}). \tag{10}$$

Having divided Eq. (8) into imaginary and real parts, we obtain

$$Y = \frac{1}{N}\sqrt{(\bar{n}_1 N - 1)(1 - N/\bar{n}_1)} \quad (\text{TP}), \quad Y = \frac{1}{N}\sqrt{(\bar{n}_1 N/n_1^2 - 1)(1 - n_1^2 N/\bar{n}_1)} \quad (\text{PP}), \tag{11}$$

$$\tan 4\pi x = \frac{2NY}{1 - N^2(1 + Y^2)}. \tag{12}$$

It is known that when a nonabsorbing substrate is used in the two-layer system considered, the conditions of its clarification hold if the coating layer thickness is a multiple of $\lambda_{1d}/4$ irrespective of the type of polarization of the electromagnetic radiation incident at an angle [5]. At the same time, reflectionless passage of a PP wave through such a nonabsorbing two-layer system is also possible at the Brewster angle. The latter may occur in the absence of coating or when its thickness is multiple of $\lambda_{1d}/2$. However, if there is an absorbing substrate in the system considered, then, when the coating thickness is selected appropriately, the conditions of reflectionless quenching of incident radiation in it may be satisfied just as at a certain clarification angle of the incidence of TP and PP waves as at the angle of incidence of a PP wave, which is an analog of the Brewster angle for transparent media.

For the possible realization of both variants we assume that the reflectionless quenching of a wave in a substrate takes place when the coating thickness is close to values that are multiples of $\lambda_{1d}/4$ or $\lambda_{1d}/2$, respectively, at the clarification and Brewster angles:

$$x = \frac{l}{\lambda_{1d}} = \frac{2m-1}{4} + \Delta\,, \tag{13}$$

$$x = \frac{l}{\lambda_{1d}} = \frac{m}{2} + \Delta\,. \tag{14}$$

In both variants of reflectionless absorption of a wave the parameter Δ entering into (13) and (14) in the case of an absorbing substrate differs from zero and is determined from the joint solution of Eqs. (12) and (13) or (12) and (14):

$$\Delta = \frac{1}{4\pi} \arctan \frac{2NY}{1 - N^2 (1 + Y^2)}\,. \tag{15}$$

In the particular case of normal incidence of a wave upon the two-layer system considered $\alpha_0 = 0$, $p = 0$, $\bar{n} = n$, $\bar{y} = y$, and $\bar{n}_1 = n_1$. Then $N = n/n_1$, $Y = y$, $A = 1/n_1$ and Eqs. (11) and (12) can be presented in the form

$$y = \frac{1}{n}\sqrt{(n-1)\,(n_1^2 - n)}\,, \quad \tan 4\pi x = \frac{2nn_1 y}{n_1^2 - n^2\,(1 + y^2)} \tag{16}$$

and they coincide with the equations obtained in [1].

Equations (11)–(15) determine the conditions of reflectionless absorption of electromagnetic radiation during its incidence at an angle on the coating–absorbing substrate system. They were used to find the dependences (corresponding to these conditions) between selective values of ε' and ε'' of the substrate substance, ε_1 and l of the coating substance, the radiation wavelength λ, and the wave incidence angle α_0.

As an example, Fig. 1 presents the dependences of ε'' on ε' and l/λ on ε' at $n_1 = 1.5$, respectively, for the cases of incidence of transversely and parallel-polarized waves on the system. The functions $\varepsilon''(\varepsilon')$ have a shape close to a semi-circle, and the character of their behavior at different angles of incidence depends on the type of wave polarization. In the case of normal incidence of a wave upon a two-layer system ($\alpha_0 = 0$) the dependence of ε'' on ε' intersects the abscissa axis at the points $\varepsilon' = 1$ and $\varepsilon' = \varepsilon_1^2$. In the case of a TP wave, the radius of the semi-circle increases with α_0, whereas the very dependences of ε'' on ε' are displaced upwards, and at $\alpha_0 = 90^{\circ}$ go to infinity. Moreover, all the curves of the family lie above the limiting dependence for $\alpha_0 = 0$, which corresponds to the case of normal incidence of a wave. The points of intersection of the curves with the abscissa axis are determined by the conditions of clarification of a nonabsorbing substrate, and their coordinates are determined from the expression that follows from Eq. (4) at $y = 0$ and $x = (2m-1)/4$, viz.:

$$\varepsilon'_{\text{cl}} = p + \frac{(\varepsilon_1 - p)^2}{(1 - p)}\,. \tag{17}$$

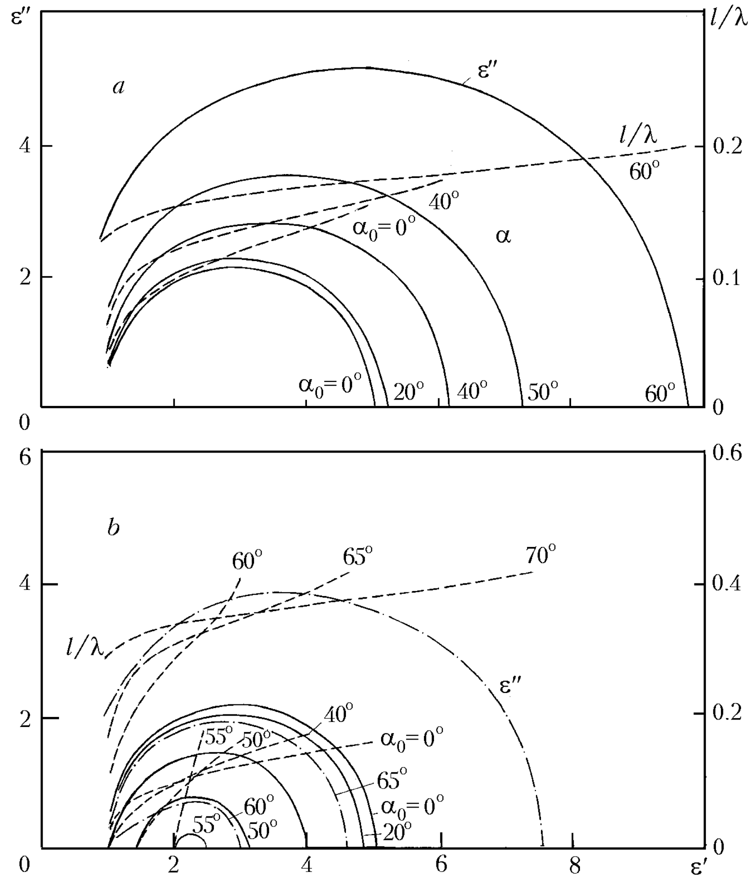

Fig. 1. Dependences of the thickness l of the coating layer and permittivity ε' on dielectric losses ε'' of the substance of absorbing substrate of infinite thickness in the course of reflectionless absorption of a TP wave incident on it at an angle α_0 (a) and a PP wave (b) in the presence of coating applied to the substrate having the refraction index $n_1 = 1.5$: solid curves — ε''; dashed curves — $1/\lambda$; dashed-dotted curves — ε'' at the Brewster angles.

In the case of a PP wave, the dependences of ε'' on ε', which approach a semi-circle shape, are displaced downwards with increasing value of α_0 and in the range 0–45° intersect the abscissa axis at the points $\varepsilon' = 1$ and $\varepsilon' = \varepsilon'_{\rm cl}$; the value of $\varepsilon'_{\rm cl}$ is determined by the conditions of clarification of a nonabsorbing substrate and can be found from the expression that follows from the second equation of (4) at $y = 0$ and $x = (2m-1)/4$:

$$\varepsilon'_{\rm cl} = \frac{1+\sqrt{1-4ap}}{2a}, \quad a = \frac{(\varepsilon_1 - p)^2}{\varepsilon_1^4(1-p)}. \tag{18}$$

When $\alpha_0 > 45^{\rm o}$, the functions $\varepsilon''(\varepsilon')$ continue to be displaced downwards with increasing α_0, but they already intersect the abscissa axis at the points $\varepsilon' = \varepsilon'_{\rm Br}$ and $\varepsilon' = \varepsilon'_{\rm cl}$, where $\varepsilon'_{\rm cl} > \varepsilon'_{\rm Br}$. The quantity $\varepsilon'_{\rm cl}$ is defined by expression (18) and the quantity $\varepsilon'_{\rm Br}$ follows from the Brewster equation $\tan\alpha_0 = \sqrt{\varepsilon'_{\rm Br}}$ that at $y = 0$ determines the existence of the wave incidence angle at which its reflectionless passage into a nonabsorbing substrate is possible in the absence of coating. With further increase in the incidence angle α_0 the functions $\varepsilon''(\varepsilon')$ are displaced downwards and at a certain boundary value of $\alpha_{\rm b}$ shrink to a point lying on the abscissa axis at $\varepsilon' = \varepsilon_1$, which follows from the condition $\varepsilon'_{\rm Br} = \varepsilon'_{\rm cl}$. On subsequent increase of α_0 the functions $\varepsilon''(\varepsilon')$ appear again, attain the limiting function that corresponds to $\alpha_0 = 0$, and at $\alpha_0 \to 90^{\rm o}$ go to infinity. These functions also intersect the abscissa axis at the points $\varepsilon' = \varepsilon'_{\rm Br}$ and $\varepsilon' = \varepsilon'_{\rm cl}$. However, in this region of variation of α_0, for the least, in value, coordinate of the intersection point there

TABLE 1. Calculated Values of the Angles of Wave Incidence α_0 and Thickness l of Coating Layer of Clarified Absorbing Substrate at Which the Conditions for Reflectionless Absorption in It of Incident Radiation Appear (n_1 = 1.5)

Substrate substance	λ	ε'	ε''	Transversely polarized wave			Parallel-polarized wave					
				Clarification angles			Clarification angles			Brewster angles		
				α_0	l at m = 1	l at m = 2	α_0	l at m = 1	l at m = 2	α_0	l at m = 1	l at m = 2
Methanol	0.818	5.35	3.20	49.7	0.14	0.45	—	—	—	62.7	0.316	0.665
Ethanol	3.20	3.85	1.05	—	—	—	36.3	0.50	1.86	64.0	1.22	2.55
Propanol	3.22	3.53	1.16	—	—	—	38.8	0.50	1.66	63.6	1.20	2.53
Butanol	3.22	3.14	0.75	—	—	—	46.7	0.49	1.72	61.8	1.19	2.51
2-Ethylpyridine	3.22	5.73	2.65	47.7	0.55	1.78	—	—	—	69.2	1.27	2.64
4-Ethylpyridine	3.22	5.76	3.95	54.6	0.56	2.25	—	—	—	70.7	1.25	2.63

corresponds $\varepsilon' = \varepsilon'_{cl}$ (Eq. (18)) and for the highest one — the expression $\varepsilon' = \varepsilon'_{cl}$ (Brewster equation), but at a coating thickness that is a multiple of $\lambda_{1d}/2$. When TP and PP waves fall in the ranges of incidence angles 0–90^o and 0–α_b, the dependences of l/λ on ε' are determined from Eq. (13). In the case of incidence of a PP wave in the range of incidence angles α_b–90^o, the behavior of these dependences is determined by Eq. (14).

For the substance of an absorbing substrate with the values of ε' and ε'' known for the given frequency of incident radiation, selective values of the wave incidence angles and the respective thickness of coatings at which there is no wave reflection can be found graphically or from Eqs. (11)–(15) at the known value of ε_1 of the clarifying coating. From Fig. 1a it follows that if the working point with such values of ε' and ε'' is located in the plane with the coordinates [ε', ε''] below the limiting dependence for α_0 = 0, then in such a substrate reflectionless absorption of a TP wave is impossible. Otherwise complete absorption of the wave occurs at a strictly defined clarification angle and at the coating thickness corresponding to it.

When a PP wave is incident at an angle, two variants of problem solution are generally possible. If the working point is located below the limiting function $\varepsilon''(\varepsilon')$ for α_0 = 0, then in the range of incidence angles 0–α_b the complete absorption of a wave is possible at a certain clarification angle of incidence and the thickness of the coating layer corresponding to it (see Fig. 1b). Complete absorption of a wave can also be observed at $\alpha_0 > \alpha_b$, but in this variant of solution for these selective values of the incidence angle there will correspond the analog of the Brewster angle realizable at selective thicknesses of the coating layer which are close to the values multiple of $\lambda_{1d}/2$. If the working point with the values of ε' and ε'' of the substance is located higher than the limiting dependence, the reflectionless absorption of a wave will take place only at one incidence angle which is analogous to the Brewster angle for a nonabsorbing substrate.

Table 1 contains selective values of the wave incidence angles and coating layer thicknesses which are calculated on the basis of Eqs. (11)–(15) and at which in the two-layer system considered conditions are created for complete absorption of the electromagnetic radiation incident on it. As a material for coating, a dielectric was used in the system with n_1 = 1.5 and as a material for the substrate — various polar liquids that possess dispersion in the range of microwaves [6]. At the tabulated values of ε' and ε'' of the liquids measured in the range of microwaves, the reflectionless absorption of waves in these liquids may be expected in the ranges of α_0 and l values 30–70^o and 0.4–4 cm, respectively.

Thus, the theoretical investigations of the character of reflection of electromagnetic radiation from an absorbing infinite layer of substance with an applied layer of a nonabsorbing dielectric points to the existence of the conditions and possibility of observing experimentally the phenomenon of complete reflectionless absorption of radiation of given frequency at strictly defined (for the materials used) coating layer thickness and incidence angles of the wave of definite polarization.

NOTATION

l, coating layer thickness, cm; m, number of zero minimum of reflected wave; n and y, refraction index and factor of dielectric losses of substrate substance; n_1, refraction index of coating substance; $\bar{n}$, $\bar{y}$, and $\bar{n}_1$, reduced values of the optical coefficients of the substrate and coating; Z_0, Z_1, and Z, wave resistances of vacuum, substance of coating, and of substrate; Z_{in}, inlet resistance of a two-layer coating–substrate system; α_0, angle of wave incidence, deg; α_1 and α_2, angle of wave refraction in the material of coating and substrate, deg; Δ, the magnitude of deviation of the coating layer thickness from the values multiple of the quarter-wavelength in the coating substance; δ and $\bar{\delta}$, angle of dielectric losses of the substrate substance and its reduced value; γ, constant of wave propagation in the coating substance; ε' and ε'', permittivity and dielectric losses of substrate substance; ε_1, permittivity of the coating substance; $\bar{\varepsilon}'$, $\bar{\varepsilon}''$, and $\bar{\varepsilon}_1$, reduced values of dielectric coefficients of substrate and coating; λ, λ_d, and λ_{1d}, wavelength in vacuum, in the substances of substrate and coating, cm; $\bar{\lambda}$, $\bar{\lambda}_d$, and $\bar{\lambda}_{1d}$, reduced values of wavelength in vacuum, in the substances of substrate and coating, cm; $\hat{\rho}$, complex value of the reflection coefficient of a wave. Subscripts: Br, analog of the Brewster angle; b, boundary value; d, 1d, dielectric substrate and coating; cl, clarification.

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